International Journal of Heat and Mass Transfer 52 (2009) 4203–4210

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

A new comprehensive model for nucleate pool boiling heat transfer of pure liquid at low to high heat fluxes including CHF

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ARTICLE INFO

Article history:
Received 18 November 2008
Received in revised form 8 April 2009
Accepted 14 April 2009
Available online 3 June 2009

Keywords:
Boiling heat transfer
Fractal dimension
Critical heat flux

ABSTRACT

Based on the fractal distribution of nucleation sites present on heating surfaces, a new comprehensive model is developed for the nucleate pool boiling of pure liquid at low to high heat fluxes including the critical heat flux (CHF). The proposed model is expressed as a function of total number, minimum and maximum sizes of active nucleation sites, fractal dimension, superheat temperature, and properties of fluids. No additional empirical constant is introduced in the proposed model. This fractal model contains less empirical constants than the conventional models. The model predictions are in good agreement with the available experimental data.

1. Introduction

The mechanisms of nucleate boiling have hotly been debated in the past decades, and the physical nature is still far from being well understood because of its complexity and diversity. The steady and strong interest is attributed to practical applications since it is desirable to design an efficient heat exchanger or boiler to operate at as high heat flux as possible without risk of physical burnout. There are many empirical correlations and models for nucleate boiling in the literature, with each applicable to a restricted range of experimental conditions. From a mechanistic viewpoint, although the influences of some parameters such as heater geometry, roughness of surfaces and contact angle, etc. have extensively been studied, a comprehensive mechanistic description is still unavailable. In addition, each model/correlation has its disadvantages because of the limitations of experiment conditions. So, searching a comprehensive theory and unified model becomes a challenging task. In the next section, some available models reported in the literature are reviewed briefly. Then, in Section 3, a comprehensive fractal model is presented for nucleate pool boiling heat transfer of pure liquid at low to high heat fluxes including the CHF. The results and discussions are shown in Sections 4 and 5 presents some conclusions from this work.

2. Some available models

In 1934, Nukiyama [1] developed a basic understanding of the physical processes that occur during boiling by heating a nichrome wire in a saturated pool of water. He distinguished different modes of pool boiling such as partial nucleate boiling, fully developed nucleate boiling, transition boiling and film boiling.

Rohsenow [2] proposed a physical model of nucleate boiling as well as a theoretical expression for heat transfer coefficient containing three empirical constants (c_{sf}, s, 0.33)

$$\frac{c_{pf}\Delta T}{h_{fg}\mathrm{Pr}^{s}} = c_{sf}\left[\frac{q}{\mu h_{fg}}\sqrt{\frac{\sigma}{g(\rho_f - \rho_g)}}\right]^{0.33} \tag{1}$$

Han and Griffith [3] subdivided the heating surface into (1) the region of bulk convection (influenced) and (2) the region of free convection (not influenced by the departing bubble). The heat flow from the surface consists of two parts: natural convection in the non-influenced and transient conduction in the influenced region. The area influenced by the bubbles was said to be a circular area having a diameter twice that of the departing bubble. In this region, Han and Griffith postulated the formation of a superheated thermal boundary layer by transient heat conduction which induces bubble formation. Therefore, the total heat flux is

$$q = q_{nc} + q_{bc} \tag{2}$$

Subsequent Mikic and Rohsenow [4] modified the Han and Griffith model by including the effect of the heating surface characteristics and proposed the functional dependence of partial nucleate boiling heat flux on wall superheat. By assuming that the contribution of evaporation to total heat removal rate is small, they obtained an expression for the partial nucleate boiling heat flux as

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doi:10.1016/j.ijheatmasstransfer.2009.04.010

Nomenclature

A	area of heating surface (m^2)
Ar	Archimedes number
C_{pf}	specific heat ($J\ kg^{-1}\ K^{-1}$)
c_{sf}	empirical constant used in Eq. (1) (dimensionless)
D	diameter (m)
D_b	bubble departure diameter (m)
D_d	the departure diameter of bubble (m)
D_f	fractal dimension
D_P	the diameter of a pore (m)
$D_{c,min}$	the minimum active cavity diameter (m)
$D_{c,max}$	the maximum active cavity diameter (m)
f	the departure frequency of bubble (s^{-1})
g	gravity acceleration ($m\ s^{-2}$)
h_{fg}	latent heat of vaporization ($J\ kg^{-1}$)
J	the average latent heat removal by per bubble ($W\ m^{-2}$)
k	thermal conductivity ($W\ m^{-1}\ K^{-1}$)
K	proportional constant for bubble diameter of influence
m	empirical constant used in Eqs. (8) and (9) (dimensionless)
N_a	active nucleation site density (m^{-2})
$N_{a,tot}$	total number of nucleation sites
$\overline{N}$	average density of active nucleation (m^{-2})
Pr	Prandtl number for fluid
q_{bc}, q_e, q_{LH}, q_{ME}	heat flux by boiling ($W\ m^{-2}$)
q_{con}, q_R	heat flux due to transient conduction ($W\ m^{-2}$)
q_{FC}	heat flux by forced convection ($W\ m^{-2}$)
q_{nc}, q_{NC}	natural convection ($W\ m^{-2}$)
q	heat flux ($W\ m^{-2}$)
q_{CHF}	critical heat flux ($W\ m^{-2}$)
r_c	radius (m)
R_c	the cavity radius (m)
s	empirical constant used in Eq. (1) (dimensionless)
T	temperature (K)
T_s	saturation temperature (K)
T_∞	bulk temperature (K)
ΔT	wall superheat ($T_w - T_s$) (K)

Greek symbols

α_f	thermal diffusivity ($m^2\ s^{-1}$)
β	cone half angle (°)
γ	$\sqrt{\frac{k_w \rho_w c_{pw}}{k_f \rho_f c_{pf}}}$
δ	thermal layer thickness (m)
θ_s	$(T_s - T_\infty)$ (K)
θ_w	$(T_W - T_\infty)$ (K)
v_e	volume of the micro/macrolayer associated with each bubble (m^3)
v_f	kinematic viscosity ($m^2\ s^{-1}$)
ρ	density ($kg\ m^{-3}$)
σ	surface tension ($N\ m^{-2}$)
τ_g	bubble growth time (s)
τ_w	bubble waiting time (s)
ϕ	contact angle (°)

Subscripts

b	bubble
c	cavity
f	liquid phase
g	gas phase
max	maximum
min	minimum
nc/NC	natural convection
s	saturation condition
tot	total
w	wall

$$q = q_{con} + q_{nc} = \frac{K^2}{2}\sqrt{\pi(k\rho c_p)_f f} D_d^2 N_a \Delta T + \left(1 - \frac{K}{4} N_a \pi D_d^2\right) h_{nc}(T_w - T_\infty) \tag{3}$$

where the parameter K is reflective of the area of influence of a bubble, and a value of 2 was assigned. N_a, D_d and f are the site density, the departure diameter and frequency of bubble, respectively. Eq. (3) did yield the experimentally observed dependence of q on ΔT. It should be noted that a quantitative prediction from Eq. (3) of dependence of heat flux on wall superheat requires knowledge of several empirical constants. Though Eq. (3) was derived for partial nucleate boiling, it was suggested that it could be extrapolated to fully developed nucleate boiling.

Judd and Hwang [5] employed an approach similar to that by Mikic and Rohsenow [4] but included micro/macrolayer evaporation at the base of the bubble as well. Thus, a third term for microlayer contribution should be added to the right-hand side of Eq. (3), and the term is

$$q_e = v_e N_a \rho_f h_{fg} f \tag{4}$$

Using the microlayer thickness measured from experiments in which dichloromethane was boiled on a glass surface, and assuming that parameter K in Eq. (3) had a value of $\sqrt{1.8}$, they were able to match the predictions with experimental data.

Recently Paul and Abdel-Khalik [6] studied the nucleate pool boiling of saturated water on a horizontal electrically heated platinum wire. From motion pictures, they determined the nucleation site density, bubble diameter at departure, and bubble release frequency. From this information, they were able to find the heat flux associated with phase change (evaporation), and the natural convection contribution was determined from single phase convection data reported in the literature. It was found that the natural convection is the dominant mode of heat transfer at low heat fluxes. At intermediate and high heat fluxes, phase change is a major contributor. Enhanced natural convection is important only in the intermediate region. Then, the total heat flux can be expressed by

$$q = q_{LH} + q_{NC} + q_{FC} \tag{5}$$

Benjamin and Balakrishnan [7] presented a new model which took into account the following mechanisms: (1) heat absorbed by the evaporating microlayer (q_{ME}), (2) heat energy expended in re-formation of the thermal boundary layer (q_R), and (3) heat transferred by turbulent natural convection (q_{NC}). The total boiling heat flux is obtained based on the above three fluxes as

$$q = \frac{q_{ME}\tau_g + q_R \tau_w}{\tau_g + \tau_w} + q_{NC} \tag{6}$$

where the weighed sum of the first two fluxes is used because the two modes are complementary to each other. But there is a parameter N_a, nucleation site density, which is based on a correlation valid only in the low to moderate heat flux regime in their model.

More recently Yu and Cheng [8] investigated the pool boiling heat transfer based on the fractal distribution of nucleation sites on boiling surfaces. Analytic expressions for the fractal dimension and area fraction of nucleation sites were derived. There are three main mechanisms contributing to nucleate boiling heat transfer: the bubble generation and departure from nucleation sites on the superheated surface, natural convection on inactive nucleation areas of the heated surface, and microlayer evaporation underneath the bubbles. The total average heat flux of the partial nucleate pool boiling heat flux can be expressed as

$$q = q_b + q_{nc} + q_{me} \tag{7}$$

Their model is applicable only to nucleate pool boiling heat transfer of pure liquid at low to moderate heat fluxes.

From the above brief survey, it is seen that a heat transfer model for pool boiling on horizontal surface, which takes into all the known mechanisms of heat transfer, is still desirable. Up to date there are few models that take into account of the size distribution of active site. In this work, we derive a new fractal model for nucleate pool boiling of pure liquid at low to high heat fluxes including CHF based on the existing models of nucleate pool boiling and on the fractal distribution of nucleation sites present on heating surfaces.

3. Description of fractal model

3.1. Correlations for predicting active nucleation site density

The density of active sites on heater surfaces may depend on the interaction among several parameters such as heater and liquid properties, distributions of cavities on heater surfaces and liquid–solid contact angles. It has been shown that the density or number N_a of active nucleate sites on heated surfaces has the significant influences on boiling heat transfer. Several investigators studied the functional dependence of N_a on q and ΔT.

Mikic and Rohsenow [4] might be the first to relate the active nucleation site density to the sizes of cavities present on the heated surfaces and expressed the functional dependence of active nucleation site density on commercial surfaces as

$$N_a \sim \left(\frac{D_{c,\max}}{D_c}\right)^m \tag{8a}$$

where $D_{c,\max}$ is the diameter of the largest cavity present on surfaces, m is an empirical constant (= 6.5) and D_c is given by

$$D_c = \frac{4\sigma T_s}{\rho_g h_{fg} \Delta T} \tag{8b}$$

Bier et al. [9], on the other hand, expressed N_a as a functional of cavity size from heat transfer data. The expression is given by

$$\ln N_a = \ln(N_{\max})\left[1 - \left(\frac{D_c}{D_{c,\max}}\right)^m\right] \tag{9}$$

where $N_{\max}$ is the value corresponding to $D_c = 0$. The value of the exponent m was found to depend on the surface preparation procedure.

Cornwell and Brown [10] made a systematic study on active nucleation site density of water boiling at 1.013 bar on a copper surface, with surface condition varying from smooth to rough, and related the dependence of active site density on wall superheat as

$$N_a \sim \Delta T^{4.5} \tag{10a}$$

They justified their observed functional dependence on wall superheat by assuming only conical cavities existed on surfaces and that vapor needed to be trapped in cavities before any nucleation could occur. They also related the cavity size to the total number of cavities presenting on the surface from the cavity size data obtained by using an electron microscope, and $N_{a,tot}$ is expressed as

$$N_{a,tot} \sim \frac{1}{D_c^2} \tag{10b}$$

Yang and Kim [11] attempted to quantitatively predict the active nucleation sites from knowledge of the size and cone angle distribution of cavities that are actually present on the surface. Using a scanning electron microscope and a differential inference contrast microscope, they established the dependence of the nucleation site density on the characteristic of a boiling surface with the aid of statistical analysis approach. They used Bankoff's [12] criteria to determine which cavities will trap gas. This condition was given by $\phi > 2\beta$. By combining the probability distribution function and this criterion, they related N_a to the average $\overline{N}$ on the surface as

$$N_a = \overline{N}\int_{R_{\min}}^{R_{\max}} \lambda e^{-\lambda r}\,dr \int_0^{\phi/2} \frac{1}{\sqrt{2\pi s}} \exp\left[-\frac{(\beta-\bar{\beta})^2}{2s}\right] d\beta \tag{11}$$

where $\bar{\beta}$ is the mean value of cone half angle, λ and s are statistical parameters. These parameters are dependent upon the surface preparation procedure and the material of surface.

Kocamustafaogullari and Ishii [13] developed a relation for active nucleation site density in pool boiling. Their correlation related the active nucleation site density to the dimensionless minimum cavity size and density ratio. The correlation for system pressures from 1.0 to 198.0 bar is

$$N_a^* = f(\rho^*) r_c^{*-4.4} \tag{12}$$

where $N_a^* = N_a D_d^2$, $r_c^* = 2r_c/D_d$, $r_c = 2\sigma T_s/\rho_f h_{fg}\Delta T$, $\rho^* = (\rho_f - \rho_g)/\rho_g$, $D_d = 0.0012(\Delta\rho/\rho_g)^{0.9} D_{dF}$, $f(\rho) = 2.157\times 10^{-7}\rho^{*-3.2}(1+0.049\rho^*)^{4.13}$ is density function, and D_{dF} is the Fritz [14] diameter given by $D_{dF} = 0.0208\phi[\sigma/g(\rho_f - \rho_g)]^{1/2}$.

Jakob and Linke [15] reported the relationship between Na and q. However, his observations were limited to the cases of low heat flux. Gaertner and Westwater [16] employed a novel technique in which nickel salts were dissolved in water and the heater surface acted as one of the electrodes. By counting the numbers of holes in the deposited layer, they found the functional dependence of active nucleation site density on wall heat flux to be

$$N_a \sim q^{2.1} \tag{13}$$

Paul and Abdel-Khalik [6] conducted their experiments on the pool boiling of saturated water at 1 atm along an electrically heated horizontal platinum wire. Using high-speed photography, they measured active nucleation site density and bubble departure diameter. They found that the active nucleation site density of N_a can be represented by the linear relationship with the boiling heat flux as follows:

$$N_a = 1.207\times 10^{-3} q + 15.74 \tag{14}$$

Wang and Dhir [17] performed a systematic study of the effect of contact angle on the density of active nucleation sites. The correlated cavity size D_c was related to the wall superheat for nucleation as given by Eq. (8b). For surfaces with $46^\circ < \phi < 60^\circ$, N_a and D_c are correlated by

$$N_a = 5.0\times 10^5 (1-\cos\phi) D_c^{-6} \tag{15}$$

It is seen that so far the available models for N_a versus q are usually correlated with several empirical constants which lack special physical meanings, and the mechanisms behind these constants are still not clear. Therefore, a complete mechanistic description for N_a and q is desirable. The next section focuses on the description of N_a based on the fractal distribution of nucleation sites present on heated surfaces.

3.2. Fractal analysis of nucleation sites on a boiling surface for pool boiling

According to Yu and Cheng [8], the active cavities on the heated surface are analogous to pores in porous media and the cumulative number N_a of active cavities with diameters greater than and equal to D_c can be described by

$$N_a(D_L \geqslant D_c) = (D_{c,\max}/D_c)^{D_f} \quad \text{with} \quad D_{c,\min} \leqslant D_c \leqslant D_{c,\max} \tag{16}$$

Eq. (16) implies that the cavities acting as bubble emitting centers actually are statistically self-similar in the range of minimum diameter $D_{c,\min}$ and maximum diameter $D_{c,\max}$.

The number of active cavities of sizes lying between D_c and $D_c + dD_c$ can be obtained from Eq. (16) as

$$-dN_a = D_f D_{c,\max}^{D_f} D_c^{-(D_f+1)} dD_c \tag{17}$$

where $dD_c > 0$ and $-dN_a > 0$, implying that the number of active cavities decreases with the increase of cavity sizes.

The total number of nucleation sites from the minimum active cavity to the maximum active cavity can be obtained from Eq. (16) as

$$N_{a,tot} = \left(\frac{D_{c,\max}}{D_{c,\min}}\right)^{D_f} \tag{18}$$

The minimum active cavity diameter $D_{c,\min}$ and the maximum active cavity diameter $D_{c,\max}$ could be predicted by Hsu's model [18]:

$$D_{\min} = \frac{\delta}{C_1}\left[1 - \frac{\theta_s}{\theta_w} - \sqrt{\left(1 - \frac{\theta_s}{\theta_w}\right)^2 - \frac{4\zeta C_3}{\delta\theta_w}}\right] \tag{19a}$$

$$D_{\max} = \frac{\delta}{C_1}\left[1 - \frac{\theta_s}{\theta_w} + \sqrt{\left(1 - \frac{\theta_s}{\theta_w}\right)^2 - \frac{4\zeta C_3}{\delta\theta_w}}\right] \tag{19b}$$

where $\zeta = \frac{2\sigma T_s}{\rho_g h_{fg}}$, $C_1 = \frac{(1+\cos\phi)}{\sin\phi}$ and $C_3 = 1 + \cos\phi$, with ϕ being the contact angle of the fluid and the heater material, δ is the thermal boundary layer thickness which can be usually expressed as

$$\delta = \frac{k_f}{h_{nc}} \tag{20}$$

where h_{nc} is the average heat transfer coefficient for natural convection. For natural convection, Han and Griffith [3] applied the following correlation:

$$h_{nc} = 0.54\rho_f c_{pf}\left[\frac{\gamma g(T_w - T_\infty)\alpha_f^3}{\sqrt{A}\upsilon}\right]^{1/4} \tag{21a}$$

For turbulence, Han and Griffith [3] applied the following correlation:

$$h_{nc} = 0.14\rho_f c_{pf}\left[\frac{\gamma g(T_w - T_\infty)\alpha_f^2}{\upsilon}\right]^{1/3} \tag{21b}$$

If Eq. (21) is substituted into Eq. (20), the thermal boundary layer δ can be obtained from Eq. (20). Hsu [18] pointed out that a cavity can be active in the range of $D_{c,\min} \leqslant D \leqslant D_{c,\max}$. A cavity can be ineffective at low wall temperature (or low heat flux).

In nucleate pool boiling, the fractal dimension D_f of nucleation sites is given by Yu and Cheng [8] as

$$D_f = \frac{\ln\left[\frac{1}{2}\left(\frac{\overline{D}_{c,\max}}{D_{c,\min}}\right)^2\right]}{\ln\frac{D_{c,\max}}{D_{c,\min}}} \tag{22}$$

where $\overline{D}_{c,\max}$ is the averaged value over all the maximum active cavities as

$$\overline{D}_{c,\max} = \frac{1}{(T_w - T_s)}\int_{T_s}^{T_w} D_{c,\max}(T_w)\,dT_w = \frac{1}{\Delta T}\sum_{j=1}^{m} D_{c,\max}(T_{w_j})\delta T_w = \frac{1}{m}\sum_{j=1}^{m} D_{c,\max}(T_{w_j}) \tag{23}$$

where $m = \Delta T/\delta T_w$, and a constant δT_W is assumed. In the above equation, $T_{w_j} = T_s + j(\delta T_w)$ with $j = 1, 2, \ldots, m$. For example, if we choose $\delta T_w = 0.2$ K, then $m = 5$ for $\Delta T = 1$ K, and $m = 50$ for $\Delta T = 10$ K.

Eq. (22) denotes that the fractal dimension D_f of nucleation sites is dependent upon $D_{c,\max}$ and $D_{c,\min}$ given by Eq. (19), which is a function of wall superheat and contact angle. Yu and Cheng [8] confirmed that the nucleation sites on heated surface follow the fractal scaling law and studied the fractal dimension D_f versus the contact angle, the wall superheat in the range of $\Delta T \leqslant 20$ K.

3.3. Fractal model

In our model, the total heat flux in nucleate boiling is assumed to be contributed by the following mechanisms:

(i) Latent heat by bubbles (q_{LH}) because of the evaporation of liquid. This was first suggested by Moore and Mesler [19] based on their observation of rapid surface-temperature fluctuations in nucleate boiling.
(ii) Transient conduction (q_{CON}) and subsequent replacement of the superheated liquid layer in contact with the heating surface, as proposed by Han and Griffith [3].
(iii) Heat transferred by natural convection (q_{NC}) which was discussed by Yu and Cheng [8] in their model.

The total boiling heat flux is obtained from the above three contributions as

$$q_{tot} = (q_{LH}\cdot\tau_g + q_{CON}\cdot\tau_w)f + q_{NC} \tag{24}$$

where f is the bubble departure frequency. Each bubble cycle consists of a growth period (τ_g) and a waiting period (τ_w). Eq. (24) takes into account the latent heat during the growing time and transient conduction during the waiting time. The bubble departure frequency, f, is usually expressed as

$$f = \frac{1}{\tau_w + \tau_g} \tag{25}$$

In pure liquids, Van Stralen et al. [20] assumed that the waiting time is related to the growth time by

$$\tau_w = 3\tau_g \tag{26}$$

Han and Griffith [3] obtained the analytical expression for the bubble waiting time, τ_w, which is related to the cavity size ($D_c = 2R_c$) by

$$\tau_w = \frac{9}{4\pi\alpha_f}\left[\frac{(T_w - T_\infty)R_c}{T_w - T_s(1 + 2\sigma/R_c\rho_g h_{fg})}\right]^2 \tag{27}$$

where R_c is the cavity radius. Wang and Dhir [17] measured $R_c = 1.1$–27.7 μm for pool boiling of saturated water at 1 atm pressure on a copper surface. A rough estimation of the term $2\sigma/R_c\rho_g h_{fg}$ gives 0.1–0.01 for $R_c = 1.0$–10 μm. So in Eq. (27) the term $2\sigma/R_c\rho_g h_{fg}$ can be neglected for the simplicity of integration, and Eq. (27) can be reduced to

$$\tau_w = \frac{9}{16\pi\alpha_f}\left(\frac{T_w - T_\infty}{\Delta T}\right)^2 D_c^2 \tag{28}$$

Eq. (28) indicates that the larger the active cavity, the longer the waiting time, which is consistent with the physical phenomena. It should be noted that in Eq. (28) the bulk temperature should not be set to equal the saturation temperature. Substituting Eqs. (28) and (26) into Eq. (25) for the bubble frequency, f, we can see that the bubble departure frequency, f, is related to the size of active cavities. Haider and Webb [21] also discussed the three components in their model based on Mikic and Rohsnow' model [4].

3.3.1. Latent heat component

The latent heat produces during the growth time (τ_g) when a bubble grows up by the vaporization of the thermal boundary. The latent heat flux per unit projected area can be expressed by

$$q_{LH} = N_a \frac{1}{\tau_g} J \tag{29}$$

where N_a is the average number of active nucleation sites per unit area on heating surface, J is the average latent heat removal by per bubble. This component is expected to dominate at high heat fluxes. Eq. (29) also implies that the size of each active nucleation site is uniform.

The heat flux removed by a single bubble can be obtained by

$$J = h_{fg}\rho_g V_b = \pi h_{fg}\rho_g D_b^3/6 \tag{30}$$

where D_b is the average bubble departure diameter per nucleation site, which is given by [4]

$$D_b = 0.25\sqrt{\frac{\sigma}{g(\rho_f - \rho_g)}}\left[1 + \left(\frac{Ja}{Pr}\right)\cdot\frac{1}{\mathrm{Ar}}\right]^{1/2} \tag{31}$$

where Ja is the Jacob number given by

$$Ja = \rho_f c_{pf} T_w/(\rho_g h_{fg}) \tag{32}$$

Since the nucleation site sizes are non-uniform and follow the fractal power law, based on Eqs. (17) and (29) the heat transferred by the nucleation sites (through bubbles) between D_c and $D_c + dD_c$ can be written as

$$dq_{LH} = -J\frac{1}{\tau_g} dN_a \tag{33}$$

where J, $(-dN_a)$, τ_g are given by Eqs. (30), (17), and (26), respectively. The total heat transferred by all nucleation sites from the minimum site $D_{c,\min}$ to the maximum site $D_{c,\max}$ can be obtained by

$$\begin{aligned} q_{LH} &= \int dq_{LH} = -\int_{D_{c,\min}}^{D_{c,\max}} J\frac{1}{\tau_g} dN_a \\ &= \int_{D_{c,\min}}^{D_{c,\max}} J\frac{16\pi\alpha_f}{3}\left(\frac{\Delta T}{T_w - T_\infty}\right)^2 D_c^{-2} D_f D_{c,\max}^{D_f} D_c^{-(D_f+1)}\, dD_c \\ &= J\frac{D_f}{D_f+2}\frac{16\pi\alpha_f}{3}\left(\frac{\Delta T}{T_w - T_\infty}\right)^2 D_{c,\max}^{-2}\left[(N_{a,tot})^{1+2/D_f-1}\right] \end{aligned} \tag{34}$$

Eq. (34) denotes that the latent heat flux is a function of wall superheat, fractal dimension, contact angle and physical properties of fluid. There is no new empirical constant introduced in Eq. (34). Compared to the model by conventional method, Eq. (34) takes into account the effect of distribution of nucleation site sizes through fractal dimension. It is worth pointing out that in Eq. (34) τ_g is related to τ_w through Eq. (26), while τ_w is determined by Eq. (28), which is obtained by neglecting the term $2\sigma/R_c\rho_g h_{fg}$ in Eq. (27). For other different fluids and at higher absolute values of Ts, this term may not be allowed to be neglected. In this situation, a numerical integration of Eq. (34) may be needed for q_{LH}.

3.3.2. Transient conduction component

Once a bubble departs from a nucleation site, fresh liquid comes into contact with the heating surface. Assuming only pure conduction to the liquid in the active area during the waiting period, this mechanism may be modeled as transient conduction to a semi-infinite medium (the liquid in this case) with a step change in temperature ($\Delta T = T_w - T_s$) on a surface.

Assuming that the area of influence is a $\left(a = 4\frac{\pi D_b^2}{4} = \pi D_b^2\right)$ and that the areas of influence of neighboring bubbles do not overlap, Benjamin and Balakrishnan [7] obtained the average the transient conduction of per unit projected area as

$$q_{CON} = 2\sqrt{\frac{k_f\rho_f c_{pf}}{\pi\tau_w}}(N_a\cdot a)\Delta T \tag{35}$$

Eq. (35) implies that the size of each nucleation site is the same. However, in fact, the sizes of nucleation sites are different, and therefore Eq. (35) may introduce deviations compared to real heat flux.

Since this work assumes that the sizes of nucleation sites follow the fractal scaling law, during the waiting period, the total heat transferred by all nucleation sites from the minimum site $D_{c,\min}$ to the maximum site $D_{c,\max}$ can be obtained by modifying Eq. (35) as

$$\begin{aligned} q_{CON} &= \int dq_{CON} = -\int_{D_{c,\min}}^{D_{c,\max}} 2\sqrt{\frac{k_f\rho_f c_{pf}}{\pi\tau_w}}(a)\Delta T\, dN_a \\ &= 2a\sqrt{\frac{k_f\rho_f c_{pf}}{\pi}}\Delta T\int_{D_{c,\min}}^{D_{c,\max}} \tau_w^{-1/2} D_f D_{c,\max}^{D_f} D_c^{-(D_f+2)}\, dD_c \\ &= \frac{8}{3}a\frac{D_f}{D_f+1}\sqrt{\pi k_f\rho_f c_{pf}\alpha_f} \\ &\quad\times\frac{(\Delta T)^2}{T_w - T_\infty} D_{c,\max}^{-1}\left[(N_{a,tot})^{1+1/D_f} - 1\right] \end{aligned} \tag{36}$$

Compared to Eq. (35) by convention method, Eq. (36) indicates that the heat flux by transient conduction is strongly depends on the total number (with the exponent greater than 1, which is related to the fractal dimension) of nucleation sites, and that the heat flux inversely proportional to the maximum diameter of nucleation site.

3.3.3. Natural convection component

In the area of the natural convection, heat is supposed to be transferred from heating surface into the main body of fluid by usual convection process in continuous manner. The heat flux from inactive nucleation areas ($1 - K\cdot N_a\cdot a$) of the heating surface is given by Mikic and Rohsenow [4] as

$$q_{NC} = (1 - K\cdot N_a\cdot a)h_{nc}(T_w - T_\infty) \tag{37}$$

where K is the proportional constant for bubble diameter of influence which is taken to be 2 by Dhir [22] and by Mikic and Rohsenow [4] or 1.8 by Judd and Hwang [5]. In this paper, $K = 2$ is taken, and h_{nc} is given by Eq. (22).

In nucleate pool boiling, the fractal model for natural convection is given by Yu and Cheng [8]

$$q_{NC} = h_{nc}\left[1 - K\left(\frac{D_{c,\min}}{D_{c,\max}}\right)^{2-D_f}\right]\cdot(T_w - T_\infty) \tag{38}$$

Inserting Eqs. (34), (36), and (38) into Eq. (24), we obtain a fractal model for the total wall heat flux as

$$\begin{aligned} q_{tot} &= (q_{LH}\cdot\tau_g + q_{CON}\cdot\tau_w)f + q_{NC} \\ &= J\frac{D_f}{D_f+2}\frac{4\pi\alpha_f}{3}\left(\frac{\Delta T}{T_w - T_\infty}\right)^2 D_{c,\max}^{-2}\left[(N_{a,tot})^{1+2/D_f} - 1\right] \\ &\quad + 2a\frac{D_f}{D_f+1}\sqrt{\pi k_f\rho_f c_{pf}\alpha_f} \\ &\quad\times\frac{\Delta T^2}{T_w - T_\infty} D_{c,\max}^{-1}\left[(N_{a,tot})^{1+1/D_f} - 1\right] \\ &\quad + h_{nc}\left[1 - K\left(\frac{D_{c,\min}}{D_{c,\max}}\right)^{2-D_f}\right]\cdot(T_w - T_\infty) \end{aligned} \tag{39}$$

where $D_{c,\min}$ and $D_{c,\max}$ are given by Eq. (19), h_{nc} is given by Eq. (21), D_f is obtained from Eq. (22), $N_{a,tot}$ is calculated by Eq. (18). Eq. (39) depicts that the total wall heat flux is a function of wall superheat, fractal dimension, the total number of nucleation sites, the minimum and maximum active cavity sizes, the contact angle and physical properties of fluid, and no additional/new empirical constant is introduced in this model. It indicates that Eq. (39) has less empirical constants than conventional models, and every parameter in Eq. (39) has clear physical meaning. This fractal model has taken into account all the possible mechanisms. But in conventional models some studies ignored the contribution by q_{LH}. Early studies on the mechanism of nucleate boiling heat transfer contended that the latent heat transported by vapor bubbles contributes only a few percent of the total heat flux. In this model, q_{LH}, q_{CON} and q_{NC} are calculated directly and may be applicable for nucleate pool boiling heat transfer of pure liquid at low to high heat fluxes, whereas most of conventional models are applicable only at some range of heat fluxes.

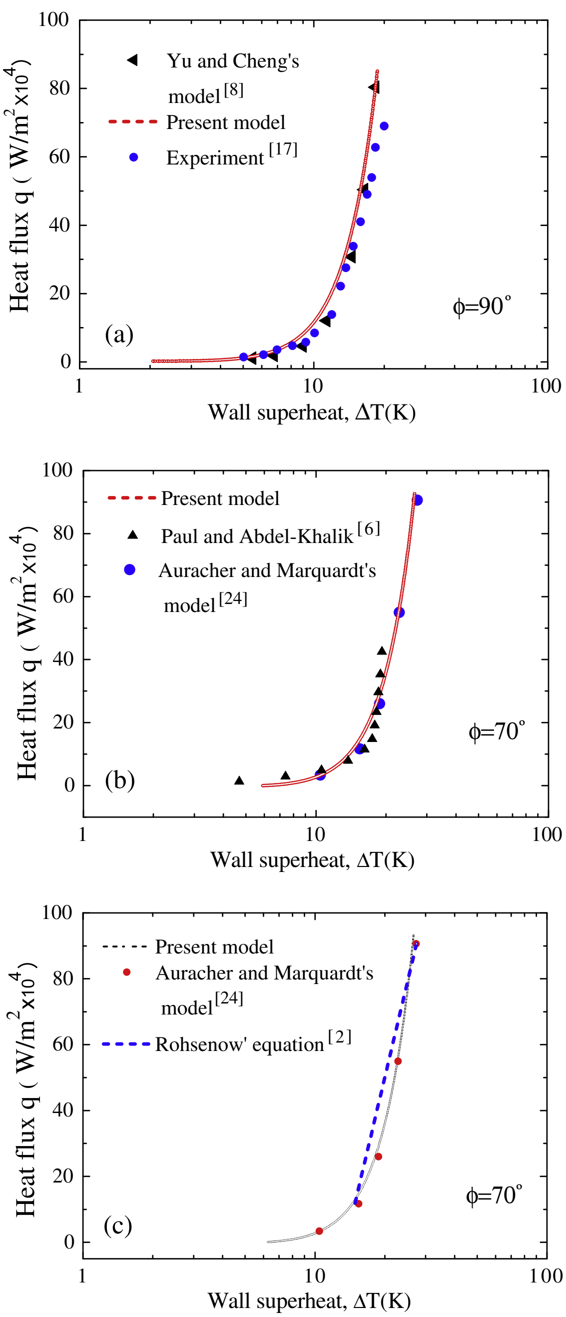

Fig. 1. A comparison between the present model and other models (or experimental data) at contact angle $\phi = 90°$. (See above-mentioned references for further information.)

4. Results and discussions

We now compare the results obtained from the present fractal model Eq. (39) with the experimental results for contact angle $\phi = 90°$ by Wang and Dhir [17] and Yu and Cheng's [8] model. The solid line in Fig. 1(a) represents the predictions by the present fractal model with the value of D_f computed from Eq. (22). An excellent agreement is found between our model predictions and Wang and Dhir's experimental data [17] from low to high heat fluxes, and this model is also in good agreement with Yu and Cheng's model at moderate heat flux. Fig. 1(b) and (c) shows a comparison among the present model predictions, experimental data and other models. It is shown that the present model predictions are found to be in good agreement with them. This indicates that our model is reasonable.

Contact angle is an important parameter affecting the bubble–wall interaction. Fig. 2 shows the effects of contact angle on the total nucleate pool boiling heat flux of fluid at 1 atm and at a wall superheat of 12 K (before CHF). It is shown that there is a large reduction in the total heat flux as the contact angle decreases from $\phi = 90°$ to $\phi = 70°$. With the help of Eq. (19) and analysis by Yu and

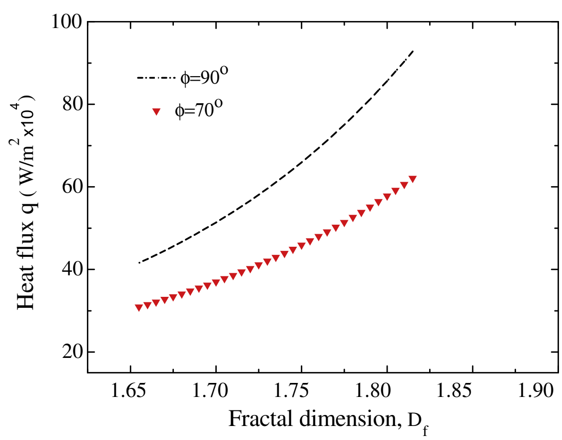

Fig. 2. Effects of contact angle and fractal dimension on nucleate pool boiling heat flux.

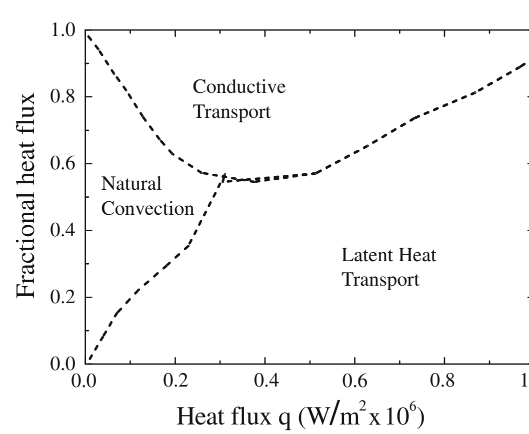

Fig. 3. The relative contribution of the latent heat flux, the conduct heat flux and the natural convection heat flux to the total heat flux calculated from Eq. (39).

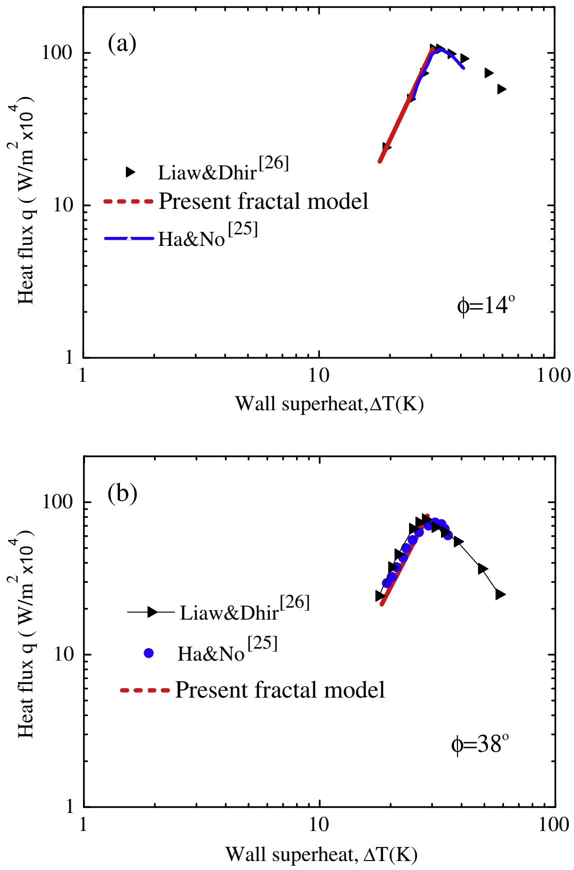

Fig. 4. A comparison between the fractal model predictions and experimental data at different contact angles.

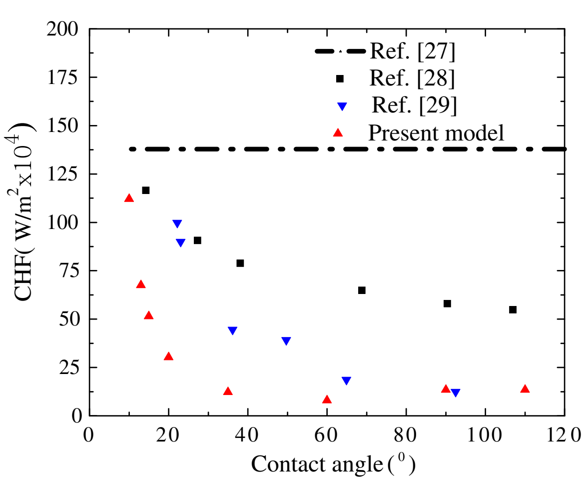

Fig. 5. Effect of contact angle on CHF for water boiling.

Cheng [8], we know that the value of $N_{a,tot}$ decreases as the contact angle decreases from $\phi = 90°$ to $\phi = 70°$, and the decrease in the value of $N_{a,tot}$ leads to a drastic reduction in heat flux. The tendency of our results is consistent with that by Wang and Dhir's [17] observations of nucleate pool boiling heat transfer. Detailed discussions were given by Yu and Cheng [8].

Fig. 3 shows the relative contribution of the latent heat flux, the conduct heat flux and the natural convection heat flux to the total heat flux. Natural convection contribution, which is the key contribution at lower superheats, gradually decreases and finally becomes zero at moderate heat flux. Heat transfer contribution due to evaporation (latent heat flux) and transient conduction increases with superheat. The latter reaches a maximum of 50% and deceases while the former continues to increase. In this case, the contributions from latent heat flux and transient conduction are more or less equal. Moreover, the contribution of the latent heat flux approaches 90% of the total flux at high heat flux. From Fig. 3, we can see that the contribution of the latent heat flux increases with the heat flux (or wall superheat). At high heat flux, the latent heat flux dominates the total heat flux. Indeed, at the critical heat flux, all the heat transferred from the surface is due solely to the latent heat flux. The contributions from the transient conduction and the natural convection tend to zero because at high heat flux (or wall superheat) there will be hardly any waiting time for bubbles and any area available for natural convection to take place. Paul and Abel-Khalik [6] and Xiao and Yu [23] also found the same conclusion.

The CHF can be calculated by

$$q_{CHF} = q_{tot} = q_{LH} \cdot \tau_g \cdot f \tag{40}$$

where q_{LH}, τ_g and f are given by Eqs. (34), (28), and (25), respectively. Fig. 4(a) shows a comparison among the present model predictions for CHF, Han and No's experimental data [25], Liaw and Dhir's model [26] for CHF at contact angle $\phi = 14°$. The high heat flux data for contact angle $\phi = 38°$ is presented in Fig. 4(b). The highest point is CHF point. The results show that the CHF from the present model is in good agreement with Liaw and Dhir's model [26] and Han and No's experimental data [25]. It is seen from Fig. 4 that the fractal model predictions are quite satisfactory, and the predicted CHF is found to be in good agreement with the others, especially at high heat flux.

Investigators [28,29] experimentally investigated the effect by changing the equilibrium contact angle with different surfaces. Fig. 5 shows a comparison among the present model, Kutateladze correlation [27], Liaw and Dhir [28], and Hahne and Diesselhorst's [29] data about the effect of contact angle on CHF for water boiling. Although the differences are somewhat higher, the present model correctly depicts this trend of the CHF versus contact angle. Kutateladze's correlation [27] does not include the contact angle as a parameter. The discrepancy between the present model and the experimental data may be attributed to the heater orientations, such as vertical by Liaw and Dhir [28] and horizontal by Hahne and Diesselhorst [29]. In the present model, we do not consider the effects of orientation of heated surface and the movement of bubbles on boiling surface. In the future work, we may study these effects.

5. Conclusions

Based on the fractal distribution of nucleation sites on heating surfaces, a new fractal model is proposed to predict the heat transfer in nucleate boiling at low to high heat fluxes including the CHF. The present model, incorporating the well-known contributions of latent heat flux, transient conduction and natural convection, provides an improved understanding of the mechanisms of nucleate pool boiling. The prediction by the present model has been shown to be good agreement with the available experimental data. The boiling is influenced by the heater geometry, size [30], and to some extent by the size and shape of a container as well as the orientation of heated surfaces. This will be our future work.

Acknowledgment

This work was supported by the National Natural Science Foundation of China through Grant No. 10572052.

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